

# Discussion of the Two-Fermion Sector in a Unified Nonlinear Spinor Field Model with Indefinite Metric II\*

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In unified field models all observable (elementary and nonelementary) particles are assumed to be bound states of elementary unobservable fermion fields. Such models are formulated by self-regularizing higher order nonlinear spinor field equations with indefinite metric. The latter needs a careful investigation of the corresponding state space, in particular with respect to bound states. Based on preceding papers the general analysis of the state space is further developed in the framework of a relativistic energy representation in Part I. In Part II this formalism is applied to bound states of the two-fermion sector for a simple model. By direct calculation it turns out that for very heavy masses of the constituent fields bound states with positive norm and small masses are possible, *i.e.*, that the two-fermion sector allows a meaningful physical interpretation.

## 3. Two-fermion sector equations

The use of state vector representations (2.1) or (2.4) resp., requires special methods for their explicit calculation. Such methods are systematically provided by functional quantum theory, cf. [22]. In connection with the investigation of the metrical structure of the state space, in particular, a calculation method is needed which is the functional analogy to the Schrödinger representation of ordinary quantum mechanics. A corresponding method was developed in a preceding paper by Grosser *et al.* [19] as a generalization of a method for treating the anharmonic oscillator [25] to the case of unified models. With respect to details we refer to this paper. In our simplified version of a unified field model the basic equation of this method is given for the state functional (2.4) by the expression

$$E|\mathfrak{F}\rangle = \sum_{r=1}^2 \int j_\varepsilon^r(\mathbf{x}) G_{\varepsilon\alpha}^0(i \mathfrak{G}_{\alpha\beta} \cdot \nabla - m_r \delta_{\alpha\beta}) \partial_\beta^r(\mathbf{x}) d^3x |\mathfrak{F}\rangle + g \lambda_1^2 \int [j_\varepsilon^1(\mathbf{x}) - j_\varepsilon^2(\mathbf{x})] G_{\varepsilon\alpha}^0 \hat{V}_{\alpha\beta\gamma\delta} d_\beta(\mathbf{x}) d_\gamma(\mathbf{x}) d_\delta(\mathbf{x}) d^3x |\mathfrak{F}\rangle, \quad (3.1)$$

where  $\alpha \equiv (\alpha, k)$  is a combined spinor-superspinor index,  $\hat{V}$  the superspinor vertex to  $V$  and  $G^0$  and  $\mathfrak{G} := (G^1, G^2, G^3)$  the superspinor Dirac matrices and

$$d_\beta(\mathbf{x}) := \sum_{r=1}^2 [\partial_\beta^r(\mathbf{x}) + \int F_{\beta\beta'}(\mathbf{x} - \mathbf{x}') j_{\beta'}^r(\mathbf{x}') d^3x']. \quad (3.2)$$

\* References see Part I, **38a**, 1064 (1983).

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In (3.1) the limiting process to equal times was already performed according to [25]. Hence this equation can be considered as a functional analogon to the Schrödinger equation and it can be used as the starting point of our investigation. In contrast to ordinary quantum mechanics the two-fermion sector is, however, characterized not only by the particle core of two interacting elementary fermions but also by a polarization cloud of increasing numbers of fermion-antifermion pairs as can be seen from an evaluation of (3.1) with (2.4).

The investigation of the full polarization cloud is far beyond our present mathematical knowledge about such equations. Hence we can only discuss appropriate approximations. In this paper we therefore restrict ourselves to the discussion of the core

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of the two-fermion states neglecting the influence of the polarization cloud completely. The core of the two-fermion states can be described by the functional states

$$|\mathfrak{F}\rangle := \sum_{r,r'=1,2} \int \varphi_{\alpha\alpha'}^r(\mathbf{r}, \mathbf{r}') j_\alpha^r(\mathbf{r}) j_{\alpha'}^{r'}(\mathbf{r}') |0\rangle d^3r d^3r' \quad (3.3)$$



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and the corresponding “diagonal” approximation of the functional equation (3.1) then reads

$$\begin{aligned} E |\mathfrak{F}\rangle &= \sum_{r=1}^2 \int j_{\varepsilon}^r(\mathbf{x}) G_{\varepsilon x}^0(i \mathfrak{G}_{x\beta} \cdot \nabla - m, \delta_{x\beta}) \partial_{\beta}^r(\mathbf{x}) d^3x |\mathfrak{F}\rangle \\ &+ g \lambda_1^2 \int [j_{\varepsilon}^1(\mathbf{x}) - j_{\varepsilon}^2(\mathbf{x})] G_{\varepsilon x}^0 W_{x\beta\gamma\delta} \left[ \sum_{r=1}^2 F_{\beta\beta'}^r(\mathbf{x} - \mathbf{x}') j_{\beta'}^r(\mathbf{x}') \right] \left[ \sum_{r=1}^2 \partial_{\gamma}^r(\mathbf{x}) \right] \left[ \sum_{r=1}^2 \partial_{\delta}^r(\mathbf{x}) \right] d^3x d^3x' |\mathfrak{F}\rangle \end{aligned} \quad (3.4)$$

with

$$W_{x\beta\gamma\delta} := \hat{V}_{x\beta\gamma\delta} - \hat{V}_{x\gamma\beta\delta} + \hat{V}_{x\gamma\delta\beta}. \quad (3.5)$$

From this equation the set of equations for the state amplitudes  $\varphi(\mathbf{r}, \mathbf{r}')$  of (3.3) can be derived by projections. This was done in [19], and we will not repeat this here, as the representation (3.3) is not appropriate for our intended investigation. In order to obtain such an appropriate representation we apply a canonical transformation to our source operators which is defined by the relations

$$\begin{aligned} j^0(\mathbf{r}) &= 2^{-1/2} [j^1(\mathbf{r}) + j^2(\mathbf{r})]; \quad \partial^0(\mathbf{r}) = 2^{-1/2} [\partial^1(\mathbf{r}) + \partial^2(\mathbf{r})]; \\ j^x(\mathbf{r}) &= 2^{-1/2} [j^1(\mathbf{r}) - j^2(\mathbf{r})]; \quad \partial^x(\mathbf{r}) = 2^{-1/2} [\partial^1(\mathbf{r}) - \partial^2(\mathbf{r})]. \end{aligned} \quad (3.6)$$

By means of this transformation the state functional (3.3) can equivalently be written

$$\begin{aligned} |\mathfrak{F}\rangle &= \int \left[ \varphi_{\alpha}^{\alpha}(\mathbf{r}, \mathbf{r}') j_{\alpha}^{\alpha}(\mathbf{r}) j_{\alpha'}^{\alpha'}(\mathbf{r}') + \varphi_{\alpha}^{\chi}(\mathbf{r}, \mathbf{r}') j_{\alpha}^{\alpha}(\mathbf{r}) j_{\alpha'}^{\chi}(\mathbf{r}') \right. \\ &\quad \left. + \varphi_{\chi}^{\alpha}(\mathbf{r}, \mathbf{r}') j_{\chi}^{\chi}(\mathbf{r}) j_{\alpha'}^{\alpha}(\mathbf{r}') + \varphi_{\chi}^{\chi}(\mathbf{r}, \mathbf{r}') j_{\chi}^{\chi}(\mathbf{r}) j_{\alpha'}^{\chi}(\mathbf{r}') \right] |0\rangle d^3r d^3r' \end{aligned} \quad (3.7)$$

and (3.4) goes over into the equation

$$\begin{aligned} E |\mathfrak{F}\rangle &= \int j_{\varepsilon}^0(\mathbf{x}) G_{\varepsilon x}^0 [i \mathfrak{G}_{x\beta} \cdot \nabla - \frac{1}{2} (m_1 + m_2) \delta_{x\beta}] \partial_{\beta}^0(\mathbf{x}) d^3x |\mathfrak{F}\rangle \\ &+ \int j_{\varepsilon}^{\chi}(\mathbf{x}) G_{\varepsilon x}^0 [i \mathfrak{G}_{x\beta} \cdot \nabla - \frac{1}{2} (m_1 + m_2) \delta_{x\beta}] \partial_{\beta}^{\chi}(\mathbf{x}) d^3x |\mathfrak{F}\rangle \\ &+ \frac{1}{2} (m_1 - m_2) \left[ \int j_{\varepsilon}^0(\mathbf{x}) G_{\varepsilon\beta}^0 \partial_{\beta}^{\chi}(\mathbf{x}) d^3x + \int j_{\varepsilon}^{\chi}(\mathbf{x}) G_{\varepsilon\beta}^0 \partial_{\beta}^0(\mathbf{x}) d^3x \right] |\mathfrak{F}\rangle \\ &+ g \frac{\lambda_1^2}{4} \int j_{\varepsilon}^{\chi}(\mathbf{x}) G_{\varepsilon x}^0 W_{x\beta\gamma\delta} [F_{\beta\beta'}^r(\mathbf{x} - \mathbf{x}') j_{\beta'}^0(\mathbf{x}') + \Delta_{\beta\beta'}^r(\mathbf{x} - \mathbf{x}') j_{\beta'}^{\chi}(\mathbf{x}')] \partial_{\gamma}^0(\mathbf{x}) \partial_{\delta}^{\chi}(\mathbf{x}) d^3x d^3x' |\mathfrak{F}\rangle \end{aligned} \quad (3.8)$$

with  $F := F^1 + F^2$  and  $\Delta := F^1 - F^2$ . If we now project by

$$\langle 0 | \partial_v^0(v) \partial_{v'}^0(v') \quad \text{or by} \quad \langle 0 | [\partial_v^0(v) \partial_{v'}^{\chi}(v') + \partial_v^{\chi}(v) \partial_{v'}^0(v')] \quad \text{or by} \quad \langle 0 | \partial_v^{\chi}(v) \partial_{v'}^{\chi}(v'),$$

resp., we obtain with the definition of the antisymmetric function

$$\chi_{\beta\beta'}(\mathbf{r}, \mathbf{r}') := \varphi_{\beta}^{\alpha}(\mathbf{r}, \mathbf{r}') + \varphi_{\beta}^{\chi}(\mathbf{r}, \mathbf{r}') \quad (3.9)$$

the system of state equations (after changing  $\mathbf{v} \rightarrow \mathbf{r}$ ,  $\mathbf{v}' \rightarrow \mathbf{r}'$ )

$$Z_{x\beta v' \beta'}(\mathbf{r}, \mathbf{r}') \varphi_{\beta}^{\alpha}(\mathbf{r}, \mathbf{r}') - M_{v\beta v' \beta'} \chi_{\beta\beta'}(\mathbf{r}, \mathbf{r}') = 0, \quad (3.10)$$

$$\begin{aligned} Z_{v\beta v' \beta'}(\mathbf{r}, \mathbf{r}') \chi_{\beta\beta'}(\mathbf{r}, \mathbf{r}') - 2 M_{v\beta v' \beta'} \varphi_{\beta}^{\alpha}(\mathbf{r}, \mathbf{r}') - 2 M_{v\beta v' \beta'} \varphi_{\beta}^{\chi}(\mathbf{r}, \mathbf{r}') \\ - V_{v\beta v' \beta'}(\mathbf{r}, \mathbf{r}') \varphi_{\beta}^{\alpha}(\mathbf{r}, \mathbf{r}') - V_{v' \beta' v \beta}(\mathbf{r}', \mathbf{r}) \varphi_{\beta}^{\alpha}(\mathbf{r}, \mathbf{r}') = 0, \end{aligned} \quad (3.11)$$

$$Z_{v\beta v'\beta'}(\mathbf{r}, \mathbf{r}') \varphi(\mathbf{r}, \mathbf{r}') - M_{v\beta v'\beta'} \alpha(\mathbf{r}, \mathbf{r}') - W_{v\beta v'\beta'}(\mathbf{r}, \mathbf{r}') \varphi(\mathbf{r}, \mathbf{r}') - W_{v'\beta'v\beta}(\mathbf{r}', \mathbf{r}) \varphi(\mathbf{r}', \mathbf{r}') = 0, \quad (3.12)$$

where the following abbreviations are used

$$\begin{aligned} Z_{v\beta v'\beta'}(\mathbf{r}, \mathbf{r}') &:= G_{v\alpha}^0 [i \mathfrak{G}_{\alpha\beta} \cdot \nabla_{\mathbf{r}} - \frac{1}{2} (m_1 + m_2) \delta_{\alpha\beta}] \delta_{v'\beta'} \\ &+ \delta_{v\beta} G_{v'\alpha}^0 [i \mathfrak{G}_{\alpha\beta'} \cdot \nabla_{\mathbf{r}'} - \frac{1}{2} (m_1 + m_2) \delta_{\alpha\beta'}] - E \delta_{v\beta} \delta_{v'\beta'}, \end{aligned} \quad (3.13)$$

$$M_{v\beta v'\beta'} := \frac{1}{2} (m_2 - m_1) [\delta_{v\beta} G_{v'\beta'}^0 + G_{v\beta}^0 \delta_{v'\beta'}], \quad (3.14)$$

$$V_{v\beta v'\beta'}(\mathbf{r}, \mathbf{r}') := g \frac{\lambda_1^2}{2} G_{v\alpha}^0 W_{\alpha\beta\beta'} F_{\alpha v'}(\mathbf{r} - \mathbf{r}'), \quad (3.15)$$

$$W_{v\beta v'\beta'}(\mathbf{r}, \mathbf{r}') := g \frac{\lambda_1^2}{2} G_{v\alpha}^0 W_{\alpha\beta\beta'} \mathcal{A}_{\alpha v'}(\mathbf{r} - \mathbf{r}'). \quad (3.16)$$

Furthermore it is convenient to define

$$\begin{aligned} \varphi_1 &:= \varphi(\mathbf{r}, \mathbf{r}'); \quad \varphi_2 := \alpha(\mathbf{r}, \mathbf{r}'); \\ \varphi_3 &:= \varphi(\mathbf{r}, \mathbf{r}'). \end{aligned} \quad (3.17)$$

Then (3.10), (3.11), (3.12), can be written in an abbreviated notation as

$$\begin{aligned} \mathfrak{Z}\varphi_1 - \mathfrak{M}\varphi_2 &= 0, \\ -(2\mathfrak{M} + \mathfrak{B})\varphi_1 + \mathfrak{Z}\varphi_2 - 2\mathfrak{M}\varphi_3 &= 0, \\ -\mathfrak{B}\varphi_1 - \mathfrak{M}\varphi_2 + \mathfrak{Z}\varphi_3 &= 0. \end{aligned} \quad (3.18)$$

This system can be resolved with respect to  $\varphi_1$ . The resolution yields

$$\varphi_2 = \mathfrak{M}^{-1} \mathfrak{Z} \varphi_1, \quad (3.19)$$

$$\varphi_3 = (-1 - \frac{1}{2} \mathfrak{M}^{-1} \mathfrak{B} + \frac{1}{2} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z}) \varphi_1, \quad (3.20)$$

and leads to the equation

$$\begin{aligned} (2\mathfrak{Z} - \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \\ + \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{B} + \mathfrak{B}) \varphi_1 &= 0. \end{aligned} \quad (3.21)$$

The functions (3.17) are the right-hand side solutions of (3.8). The left-hand side solutions of (3.8) can be gained by an analogous procedure. Without repeating this procedure for the left-hand state functional and its projections we can simply study the left-hand solutions of (3.18). This leads to the system

$$\begin{aligned} \sigma_1 \mathfrak{Z} - \sigma_2 (2\mathfrak{M} + \mathfrak{B}) - \sigma_3 \mathfrak{B} &= 0, \\ -\sigma_1 \mathfrak{M} + \sigma_2 \mathfrak{Z} - \sigma_3 \mathfrak{M} &= 0, \\ -\sigma_2 2\mathfrak{M} + \sigma_3 \mathfrak{Z} &= 0. \end{aligned} \quad (3.22)$$

An analogous resolution procedure yields

$$\sigma_2 = \sigma_3 \mathfrak{Z}^{\frac{1}{2}} \mathfrak{M}^{-1}, \quad (3.23)$$

$$\sigma_1 = \sigma_3 [\frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} - 1], \quad (3.24)$$

and for  $\sigma_3$  the equation

$$\begin{aligned} \sigma_3 [2\mathfrak{Z} - \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \\ + \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{B} + \mathfrak{B}] &= 0 \end{aligned} \quad (3.25)$$

i.e.  $\sigma_3$  and  $\varphi_1$  are the left-hand and right-hand solutions, resp., of the operator

$$\begin{aligned} \mathfrak{D} := [2\mathfrak{Z} - \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \\ + \frac{1}{2} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{B} + \mathfrak{B}]. \end{aligned} \quad (3.26)$$

According to (2.24) the norm of a corresponding eigenstate in the representation (3.3) and the correspondig representation of the left-hand state functional  $\langle \mathfrak{S} |$  is given by

$$\begin{aligned} \langle a | a \rangle &= \langle \mathfrak{S}(a) | \mathfrak{D}(a) \rangle \\ &= \sum_{i=1}^3 \int_{\beta}^{\beta'} \sigma_i(\mathbf{r}, \mathbf{r}') \varphi_i(\mathbf{r}, \mathbf{r}') d^3r d^3r'. \end{aligned} \quad (3.27)$$

If the relations (3.19), (3.20) and (3.23), (3.24), are substituted in (3.27) this expression goes over into

$$\begin{aligned} \langle a | a \rangle &= \frac{1}{2} \langle \sigma_3 [\mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} + \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{M}^{-1} \mathfrak{Z} \\ &+ \mathfrak{M}^{-1} \mathfrak{Z} \mathfrak{M}^{-1} \mathfrak{Z} - 4 - \mathfrak{M}^{-1} \mathfrak{B}] \varphi_1 \rangle. \end{aligned} \quad (3.28)$$

In the following section we will study this norm expression as well as energy eigenvalues for bound states  $|a\rangle$ .

#### 4. Two-fermion bound states

As indicated at the beginning it is our aim to describe all real, *i.e.* physical elementary particles as bound states of elementary fermions. In particular in subquark models these elementary fermions are not allowed to occur as free observable particles; rather they have always to be restricted to occur only in bound states. This means that all elementary fermions of unified field models, namely ghost particles as well as regular particles have to be confined. A simple way to achieve such a confinement consists in giving the elementary fermions very large masses. Then the question arises: Do meaningful physical bound states exist in spite of the very large masses of the constituent fermions?

We will study this question for the two-fermion sector. Two-fermion sector bound states are to be derived as bound state solutions of (3.21) or (3.25) resp. We first show that the corresponding operator (3.26) is hermitian for bound states under confinement conditions. As  $\mathcal{Z}^+ = \mathcal{Z}$  and  $\mathcal{M}^+ = \mathcal{M}$  holds, the hermiticity of  $\mathcal{D}$  depends on the properties of  $\mathfrak{B}$  and  $\mathfrak{B}$ . It is sufficient to study the  $\mathfrak{B}$ -term as the investigation of the  $\mathfrak{B}$ -term runs along the same lines. Due to the translational invariance of (3.21) or (3.25) resp., it is first possible to split off the center of mass motion by the ansatz

$$\sigma_i(\mathbf{r}, \mathbf{r}') = \exp [i \mathbf{P} \cdot \frac{1}{2} (\mathbf{r} + \mathbf{r}')] \hat{\sigma}_i(\mathbf{r} - \mathbf{r}') \quad (4.1)$$

and

$$\varphi_i(\mathbf{r}, \mathbf{r}') = \exp [-i \mathbf{P} \cdot \frac{1}{2} (\mathbf{r} + \mathbf{r}')] \hat{\varphi}_i(\mathbf{r} - \mathbf{r}'). \quad (4.2)$$

This gives for (3.27) the expression

$$\langle a | a \rangle = \delta(\mathbf{P} - \mathbf{P}') \sum_{i=1}^3 \int \hat{\sigma}_i(\mathbf{y}) \hat{\varphi}_i(\mathbf{y}) d^3y \quad (4.3)$$

with  $\mathbf{y} := \mathbf{r} - \mathbf{r}'$ . Hence the hermiticity of  $\mathcal{D}$  needs only to be studied in the relative coordinate  $\mathbf{y}$ . In these coordinates  $\mathcal{Z} \equiv \mathcal{Z}(\nabla_{\mathbf{r}}, \nabla_{\mathbf{r}'})$  goes over into  $\hat{\mathcal{Z}}(\mathbf{P}, \nabla_{\mathbf{y}})$ . Without loss of generality we may assume  $\mathbf{P} \equiv 0$  and obtain by applying  $\hat{\mathcal{Z}}\mathfrak{B}$  to a test function  $g$  the expression

$$\hat{\mathcal{Z}}\mathfrak{B}g \equiv \hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y}) g(0) \quad (4.4)$$

*i.e.*

$$\langle f \hat{\mathcal{Z}}\mathfrak{B}g \rangle \equiv \int f(\mathbf{y})^* \hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y}) g(0) d^3y. \quad (4.5)$$

We now expand  $f^*(\mathbf{y})$  in a Taylor expansion about  $\mathbf{y} = 0$  which can be assumed to exist for bound

states. Then (4.5) yields

$$\begin{aligned} \langle f \hat{\mathcal{Z}}\mathfrak{B}g \rangle &= \sum_{n=0}^{\infty} \frac{1}{n!} \int (\mathbf{y} \cdot \nabla_{\mathbf{y}})^n f(\mathbf{y}^*) \Big|_{\mathbf{y}=0} \\ &\quad \cdot \hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y}) g(0) d^3y. \end{aligned} \quad (4.6)$$

If we assume that  $\mathfrak{B}(\mathbf{y}) = \mathfrak{B}(\mathbf{y})^+$ , then  $\hat{\mathcal{Z}}\mathfrak{B}$  is hermitian if the relation

$$\langle f \hat{\mathcal{Z}}\mathfrak{B}g \rangle \approx \int f(0)^* \hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y}) g(0) d^3y \quad (4.7)$$

holds, *i.e.* if

$$\begin{aligned} \sum_{n=1}^{\infty} \frac{1}{n!} \int (\mathbf{y} \cdot \nabla_{\mathbf{y}})^n f(\mathbf{y})^* \Big|_{\mathbf{y}=0} \\ \cdot \hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y}) g(0) d^3y \approx 0 \end{aligned} \quad (4.8)$$

is valid. Such a condition is satisfied if the masses  $m_1$  and  $m_2$  of both constituent fields are very large. Then  $\mathfrak{B}(\mathbf{y})$  or  $\hat{\mathcal{Z}}(0, \nabla_{\mathbf{y}}) \mathfrak{B}(\mathbf{y})$  are so concentrated about the origin  $\mathbf{y} = 0$  that all higher momenta in (4.6) can be neglected. The same holds for  $\mathfrak{B}$ . Hence hermiticity of  $\mathcal{D}$  is guaranteed if confinement of the elementary fermions is forced by very large masses of the constituent fields. In this case, *i.e.* for  $\mathcal{D} = \mathcal{D}^+$  we have for real energy eigenvalues  $\sigma_3 \equiv \varphi_1^*$  and the norm expression (3.28) reads

$$\begin{aligned} \langle a | a \rangle &= \frac{1}{2} \langle \varphi_1^* [\mathcal{Z} \mathfrak{M}^{-1} \mathcal{Z} \mathfrak{M}^{-1} + \mathcal{Z} \mathfrak{M}^{-1} \mathfrak{M}^{-1} \mathcal{Z} \\ &\quad + \mathfrak{M}^{-1} \mathcal{Z} \mathfrak{M}^{-1} \mathcal{Z} - 4 - \mathfrak{M}^{-1} \mathfrak{B}] \varphi_1 \rangle. \end{aligned} \quad (4.9)$$

We postpone the further evaluation of this expression and next discuss the corresponding eigenvalues.

In order to avoid all complications with the spinorial degrees of freedom we treat the corresponding scalar model for a first orientation. This model is realized by the substitutions

$$\begin{aligned} \mathcal{Z} &\rightarrow [E^2 + \mathcal{A}_{\mathbf{r}} + \mathcal{A}_{\mathbf{r}'} - \frac{1}{2} (m_1^2 + m_2^2)], \\ \mathfrak{M} &\rightarrow \frac{1}{4} (m_2^2 - m_1^2) =: \varkappa; \quad \mu := \frac{1}{2} (m_1^2 + m_2^2), \\ \mathfrak{B} &\rightarrow g \lambda_1^2 \delta(\mathbf{r} - \mathbf{r}'), \\ \lambda_1 &\rightarrow (m_2^2 - m_1^2)^{-1}, \end{aligned} \quad (4.10)$$

where  $\mathcal{Z}$  results from  $p^2 = m^2$ . With these substitutions the Fourier-transform of (3.21) then reads

$$\begin{aligned} &\left[ 2(E^2 - \mathbf{p}^2 - \mathbf{p}'^2 - \mu) \right. \\ &\quad \left. - \frac{1}{2} \frac{1}{\varkappa^2} (E^2 - \mathbf{p}^2 - \mathbf{p}'^2 - \mu)^3 \right] \tilde{\varphi}_1(\mathbf{p}, \mathbf{p}') \\ &\quad + \frac{1}{2} \frac{1}{\varkappa} k (E^2 - \mathbf{p}^2 - \mathbf{p}'^2 - \mu) \int \tilde{\varphi}_1(\mathbf{p} - \mathbf{s}, \mathbf{p}' + \mathbf{s}) d^3s \\ &\quad + k \int \tilde{\varphi}_1(\mathbf{p} - \mathbf{s}, \mathbf{p}' + \mathbf{s}) d^3s = 0. \end{aligned} \quad (4.11)$$

If center of mass coordinates are introduced by

$$z = \frac{1}{2}(\mathbf{p} + \mathbf{p}'); \quad \mathbf{q} = \mathbf{p} - \mathbf{p}' \quad (4.12)$$

the wave function can be written

$$\tilde{\phi}_1(\mathbf{p}, \mathbf{p}') = \delta(\mathbf{P} - 2z) \hat{\phi}(\mathbf{q}), \quad (4.13)$$

and with this substitution equation (4.11) reads in the rest system, *i.e.* for  $\mathbf{P} \equiv 0$

$$\begin{aligned} & \left[ 2(E^2 - \mathbf{q}^2 - \mu) - \frac{1}{2} \frac{1}{\kappa^2} (E^2 - \mathbf{q}^2 - \mu)^3 \right] \tilde{\phi}(\mathbf{q}) \\ & - \frac{1}{4} \frac{1}{\kappa} k (E^2 - \mathbf{q}^2 - \mu) \int \hat{\phi}(\mathbf{s}) d^3s \\ & - \frac{1}{2} \int \hat{\phi}(\mathbf{s}) d^3s = 0. \end{aligned} \quad (4.14)$$

The corresponding eigenvalue condition is given by the relation

$$\begin{aligned} 1 &= k \int \frac{\kappa(E^2 - \mathbf{q}^2 - \mu) + 4\kappa^2}{8\kappa^2(E^2 - \mathbf{q}^2 - \mu) - 4(E^2 - \mathbf{q}^2 - \mu)^3} d^3q \\ &=: J(E). \end{aligned} \quad (4.15)$$

We now assume that  $m_1$  and  $m_2$  tend in such a way to infinity that  $\kappa \ll 1$  holds. Then we substitute  $\mu - E^2 = a^2$  and approximate for  $a^2 > 1$  the right-hand side of (4.15) by

$$J(E) \approx -k \int \frac{\kappa}{4(\mathbf{q}^2 + a^2)^2} d^3q. \quad (4.16)$$

This approximation is justified insofar as for  $a^2 > 1$  the first term in the denominator of  $J(E)$  in (4.15) can be neglected compared with the second term. The restriction  $a^2 > 1$  does not influence the physical state space of our equation. If we had an eigenvalue  $E^2$  leading to  $a^2 < 1$ , this would mean that  $E^2 \approx \frac{1}{2}(m_1^2 + m_2^2)$ , and in this case the bound state would be suppressed in the physical world in

the same way as the elementary fermion states do, *i.e.* this state would never appear in any reaction due to its large mass.

Hence the approximation (4.16) is valid for all physical states. With this approximation  $J(E)$  can exactly be calculated and gives

$$J(E) = -\frac{k\kappa}{a} \frac{1}{2} B\left(\frac{3}{2}, \frac{1}{2}\right) \quad (4.17)$$

with  $B\left(\frac{3}{2}, \frac{1}{2}\right) > 0$ . Hence the eigenvalue equation (4.15) yields

$$E = [\mu - (k\kappa)^2 \frac{1}{4} B^2]^{1/2} = [\mu - g^2 \kappa^{-2} \frac{1}{4} B^2]^{1/2}. \quad (4.18)$$

For small  $\kappa$ -values,  $\kappa^{-2}$  becomes very large and hence can compensate the large  $\mu$ -term. Therefore it is possible to obtain bound states with low masses although the constituent masses of the bound elementary fermions are very large. It can further be shown by means of (4.9) that such states possess a positive norm, *i.e.* that they must be physical states. In our approximation equation (4.14) simply reads

$$(E^2 - \mathbf{q}^2 - \mu)^2 \hat{\phi}(\mathbf{q}) + \kappa k \int \hat{\phi}(\mathbf{s}) d^3s = 0. \quad (4.19)$$

From this equation it follows that the compensation of the large  $\mu$ -term does not result from the kinetic energy. Rather it comes from the potential term which is proportional to  $\kappa^{-1}$ . Hence the kinetic energy can be neglected in comparison with the constituent mass terms and the potential energy. If we transfer this result to the spinorial case, we may replace  $\mathcal{J}$  simply by

$$\begin{aligned} \mathcal{J} &\approx -\frac{1}{2}(m_1 + m_2) [G_{v\beta}^0 \delta_{v'\beta'} + \delta_{v\beta} G_{v'\beta'}^0] \\ &=: -\frac{1}{2}(m_1 + m_2) \mathfrak{C}. \end{aligned} \quad (4.20)$$

Then with  $\mathfrak{M} = \frac{1}{2}(m_2 - m_1) \mathfrak{C}$  we obtain from (4.9)

$$\begin{aligned} \langle a | a \rangle &\approx \frac{(m_1 + m_2)^2}{(m_2 - m_1)^2} \left\langle \varphi_1^* \left[ 3 - 4 \frac{(m_2 - m_1)^2}{(m_1 + m_2)^2} - \frac{1}{(m_1 + m_2)^2(m_2 - m_1)} g \mathfrak{C}^{-1} \mathfrak{G}^0 \mathfrak{W} \mathfrak{F} \right] \varphi_1 \right\rangle \\ &\approx \frac{(m_1 + m_2)^2}{(m_2 - m_1)^2} 3 \langle \varphi_1^* \varphi_1 \rangle > 0 \end{aligned} \quad (4.21)$$

*i.e.* under confinement conditions the norm of the bound states is positive. In addition the coupling

constant  $g$  can be fixed in a suitable way. But this possibility will be discussed in subsequent papers.